

Designing Tangible Interfaces for Mathematics Learning in Elementary School

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Abstract. Manipulatives are tools to which teachers may resort to help pupils overcome learning difficulties in elementary school Mathematics. Building on innovative information and communication technology, tangible interfaces aggregate new forms of interaction to the benefits of traditional manipulatives. Following a context-based methodology, we designed a tangible interface aimed at offering material support for the learning of fractions in Brazilian elementary schools. Our design process was based on principles derived from socio-cultural theory and user-centered design guidelines which prioritizes the analysis of context and the pragmatics of use. This process allowed us to propose an artifact which would satisfy both students' needs and product requirements, as elicited in our research.

Keywords: Mathematics learning, tangible interfaces, user-centered design, context.

1 Introduction

Mathematics teaching and learning is a matter of great concern among educators and researchers due to the very many difficulties faced by teachers and students in schools. As a strategy to deal with such difficulties, elementary school teachers have traditionally resorted to “concrete materials”, also known as *manipulatives*.

Manipulatives are physical objects designed to represent abstract mathematical concepts in some sort of concrete manner (in Math we have, for instance, the golden blocks, Cuisenaire rods and Dienes blocks [14], [15]). Psychologist Jean Piaget (1896-1980) suggests that young children can only build knowledge through “concrete operations” before they can proceed formally, using solely abstract referents [21] *apud* [25]. Manipulatives, therefore, are credited as allowing students to learn abstract concepts through physical manipulation, an experience which is potentially more interesting for children's learning than, for instance, simply observing simulations running in a computer screen [17].

Computers have been used in Education since the 80's as tools to perform simulations and support communication, just to mention two of the most

widespread usages. However, there is little consensus as to how computers can effectively enter the learning process [1]. Even the traditional shape of PCs is such that it diminishes the chances of interaction [5], [6].

Tangible user interfaces (TUIs), on the other hand, offer a new paradigm of human-computer interaction (HCI) by embedding computing power and ubiquity in physical objects [31], [4]. Thus, TUIs mix physical and digital worlds [19] and change the traditional input/output paradigm of classic computing. In education, TUIs may be a solution that brings together the advantages of physical manipulation and innovative ways of interaction, enhancing therefore the learning experience [5], [19], [24].

The main goal of this work was to propose a tangible interface aimed at supporting the learning of fractions, and to explore its use in classroom contexts. For that purpose, we followed design strategies from within a context-based and user-centered methodology.

2 Tangible Interfaces in Education

The definition of tangible interfaces is still unclear, for being touched and manipulated is not sufficient to identify such interfaces. The key point we must bear in mind is that TUIs are associated to specific meanings, in contrast with a mouse, for example, which works as an input tool with no obvious single referent [19]. In tangible interfaces, meaning is coupled with the goal of the activity carried out by and with the system [9].

In order to effectively contribute to the learning process, tangible interfaces should support children's thinking (providing scaffolding) but not do the work for them [12]. The main goal of an interface for Education is not to help users to perform faster, but to engage them in new ways of reasoning [25]. In doing so, technology should be "hidden", as the focus must be on interface interactions and their effects [22]. Thus, interfaces should direct students' attention to learning objects with minimal deviation and ambiguity [19].

With tangible interfaces, children can perhaps learn in more natural ways and use multiple senses. Knowledge is therefore more explicitly built through touching, seeing and hearing. This integration of senses may bring more fun to learning, engaging students in reflection, creation and imagination while exploring the materials [22]. TUIs also stimulate collaborative work, making exchanges easier among students. In addition, tangible interfaces can reach people with different kinds of disabilities, increasing accessibility for the impaired.

Several TUIs for Education have been developed lately, following different approaches. Some tangible interfaces are connected to personal computers, functioning also as input devices. In these cases, the input takes place through a physical object, but the output is shown in the graphical interface of a computer. Examples of such approach in Education include: *SmartStep* and *FloorMath* [28] (multimedia applications intended to help children learn and practice basic Mathematics skills); and *Tangible Interface for Collaborative Learning Environments - TICLE* [27] (platform that uses computer vision techniques to connect

concrete Mathematical games to representations in a computer screen). A second approach is totally independent of personal computers, for input and output occur in the same device. This group of TUIs may be called “digital manipulatives”, and include: *SystemBlocks* and *Flowblocks* [24] (sets of blocks designed to explore concepts of systems dynamics); *Stackables* and *Programmable Beads* [24] (sets of pieces which can be connected to form different patterns); and *Electronic Blocks* [33] (set of tangible programming elements).

According to our research, we believe that for a tangible interface to be adopted in classrooms and contribute to learning, it should attend at least to the following general aspects:

- allow collaborative use;
- provide scaffolding;
- increase user engagement;
- be accessible by children with different kinds of disabilities;
- be independent of personal computers;
- be viable in the physical space of classrooms;
- be aligned with school curriculum;
- be fairly inexpensive

The tangible interfaces we have analyzed, including those mentioned above, may be credited with some but not all of these aspects. For instance, the applications developed by [27] and [28] rely on personal computers. *SystemBlocks* and *Flowblocks* offer no scaffolding and *Beads* and *Stackables* are not accessible for visually impaired children. Finally, *Electronic Blocks* are not aligned with curricular activities in school.

In this work, we propose a tangible interface whose prototype was based on observing teachers’ and students’ contextual use of manipulatives in Brazilian elementary classrooms.

3 Methodology

When a product is developed, it is meant to be used in a particular context, by a certain group of users [11]. Instead of thinking about what users *should do* with the product, it is important to understand what they already do with similar products and to understand their activities [8] and sense-making processes in specific social contexts. We believe that the environment in which a product is to become a tool will affect the making of the product itself in its technical, physical and social aspects [11].

In the development cycle of user-centered design (as proposed by ISO 13407), a critical phase is to “*understand and specify the context of use*”, which leads to a specification of user and organizational requirements [11], and to the design of solutions. Technology is therefore designed to satisfy user needs identified in the study of the context [8].

For thinking the development of interactive devices, *Activity Theory* has been considered a good alternative to bridge the gap between theory and practice [16].

On the basis of Activity Theory, we understand interactive devices as mediators of the relations built between humans and the world. Frameworks for the development of interactive devices based on Activity Theory take into account the socio-cultural context of its use in human activity [16]. This feature makes Activity Theory suitable to guide the development of technological devices that trigger the ways of interaction made possible by ubiquitous computing [8], such as tangible interfaces.

Activity Theory also provides a way to structure and analyze the type of data we built [10], as discussed later, helping designers to model users' current practice and structure future activities [16].

3.1 Design Process

Our design process was based on principles of Activity Theory and followed the user-centered development cycle defined by ISO 13407. However, we added a phase prior to the study of context, which consisted of an analysis of possible competitors to our product.

Competitor Analysis The goal of this competitor analysis was to explore features of existing similar products and discuss what may be missing [18], with special attention to identifying their faults and weak features [32]. We analyzed two groups of products: traditional manipulatives and tangible interfaces for enhancing mathematical learning. From this study, we collected and adapted guidelines for the development of our own product.

Field Observations To understand the context for our product, we observed several elementary math classrooms in Recife (Brazil), interviewed their teachers and had informal conversations with the school staff.

The observations were non-participant and non-systematic [7]: we registered the flow of activities and contextual elements, trying not to interfere in the classroom routine.

The interviews were conducted with 12 teachers from private and public schools in Recife. They were semi-structured and focused on the use of concrete materials (such as manipulatives) in classrooms. We discussed issues regarding which materials were most often used and how, and asked teachers to list their advantages and disadvantages.

Observations and interviews allowed us to understand and describe the context of use, with special focus on concrete materials. Field observations also contributed to elicit requirements and to identify favorable aspects for the introduction of tangible interfaces in classrooms.

Experiments Having described a possible context for introducing our product, we needed to focus on a specific mathematical concept, for we believe learning is always specific to a subject matter [30]. We chose the topic of fractions, known

to be a source of great difficulties in the learning of Mathematics [2], [3]. We then organized two problem-solving sessions, in which two groups of three 4th graders were asked to work collaboratively on a set of fraction problems. The main goal of this phase was to identify at least some of the students' difficulties regarding the learning of fractions.

Qualitative Data Analysis All data were analyzed qualitatively. Data from classroom observations were structured according to Engestrom model [10] of Activity. Data from interviews were coded as hierarchical categories that we used to structure the information about the use of concrete materials in classrooms. This analysis was performed combining open, axial and selective codification [7]. The experiments were video-recorded and the transcripts were analyzed to identify the main difficulties students faced while solving the problems.

Rapid Prototyping The analyses above allowed us to propose a tangible interface to support the learning of fractions. In order to test our ideas in a simple, fast and low-cost manner, we built a low-fidelity prototype [18] using colored paper. The prototype was then used in a second experiment, as an auxiliary material to which students could resort to solve the problems about fractions.

4 Results

The discussion below refers to data built in each of the subsections above, and presents the results of our research.

4.1 Design Guidelines

Our analysis of existing educational tangible interfaces allowed the compilation of diverse features, which we take as design principles for developing this kind of product:

1. to provide multi-sensorial representations through different media channels [24], which may help supporting different learning styles [34];
2. to employ resistant materials for use by children [23];
3. to be functional even in the lack of electric power [23];
4. to offer different levels of scaffolding [23];
5. to stimulate reflection and creativity [23], [24];
6. to allow connections with students experiences and interests [24];
7. to support collaborative work [26], [29].

4.2 Contextual Elements

Our field work allowed us to specify a context for using our product: elementary schools classrooms in Brazil. When visiting schools, we identified many differences between private and public institutions in Recife. Therefore, both cases

were considered in this work. Contextual elements were analyzed according to categories from Activity Theory (community and subjects, rules, and mediating artifacts).

4.3 Justifications and Requirements

The analyses of classroom observations and interviews with teachers were instrumental for detecting opportunities for introducing our product, which we present here as “justifications” for our proposal, and for eliciting user and technical requirements. Some requirements, as seen below, are very similar to those guidelines built on the basis of the competitor analysis above.

Justifications

- **Classroom usage:** access to materials not directly available in the classroom is troublesome. Being ready at hand, materials could be used whenever needed.
- **Tangibility:** concrete materials were seen by all teachers as extremely beneficial to children’s learning in elementary levels.
- **Acceptance:** teachers and students were willing to use technological innovations in classrooms.
- **Engagement:** students usually engage intensively in activities that make use of concrete materials.
- **Product for the learning of fractions:** mathematics is seen as the subject which brings the greater problems for students and teachers, and fractions is considered a difficult topic. Teachers and students alike would very much welcome a product intended to help them with the teaching and learning of fractions.

Requirements

1. **Provide scaffolding:** there is a mix of different levels of achievement in both public and private schools, and a product for such a context must provide for various levels of scaffolding.
2. **Stimulate reasoning:** the product should make students think and reflect upon concepts, instead of simply giving answers.
3. **Connect theory and practice:** the product must help teachers to make connections between formal mathematics and students’ daily life, bringing meaningfulness to learning.
4. **Be adequate to school curriculum:** products to be used in classrooms must connect to concepts in school curriculum, so that teachers can use them effectively during classes.
5. **Allow multiple activities:** the product should be flexible, so that teachers and students can create diverse activities and use it for different purposes.

6. **Support autonomous activities:** in most classes teachers can not give individual attention to students. Therefore, the product should allow students to do at least some part of the work on their own, being supervised by the teacher when possible.
7. **Allow collaborative use:** grouping students is a common and well-accepted practice in Brazilian schools, specially suitable when activities involve concrete materials. The product must then support diverse levels of collaborative work.
8. **Provide multiple representations:** to include students with disabilities in regular classroom activities, the product must provide multiple representations of the same concept, using different media.
9. **Have simple interface:** a complex interface may cause difficulties of use, causing teachers to spend greater amounts of time dealing with the product and with students during activities. Such problems may cause frustration and rejection of the product. Besides, some young children may have inaccurate motor coordination and need a simple interface to successfully interact with the product.
10. **Be resistant:** being meant for children, the product must be made of material that is durable and does not appear fragile to its users.
11. **Have low cost:** most Brazilian schools, specially those in the public system, have limited resources available. A low-cost product would have more chances to be adopted in schools.
12. **Be safe for users:** products designed for children must follow legal restrictions to prevent accidents.
13. **Provide audio control:** classes in elementary school are usually very noisy. If audio is used in the product, it must be gradable in order to adapt to different levels of ambient noise.
14. **Resist to the wind:** if the equipment is to be used outdoors or in classes with fans, it must be heavy enough to resist to the wind. Other details such as this should be considered in an actual list of requirements.

4.4 Students' difficulties with fractions

From the analysis of the problem-solving session, we identified some students' difficulties regarding the concepts of fractions. In this section we focus on the idea of fractions as a part-whole relation, explored through the manipulation of areas in geometric figures.

When studying fractions in this approach, a crucial step is to divide the figures in same-size parts. In one of the problems, illustrated below, students were asked to divide an hexagon in three equal parts. In order to make this problem harder, we presented the figure with a horizontal line dividing the figure in two halves. To get this problem right, students should ignore this middle line and then find the adequate partition. After several trials in a separate sheet of paper (as shown in Fig. 1(a)), they reached an approximate division (Fig. 1(b)) but were not able to reproduce this division in a handout (Fig. 1(c)). Similar problems occurred when students were asked to divide a circle in thirds.

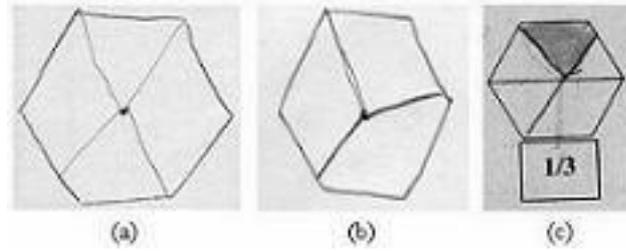


Fig. 1. Students' attempts at dividing an hexagon in three equal parts.

In another case, students had to paint half of a square, shown with its two diagonals. None of the students noticed that the diagonals already marked divisions in half. They ignored these lines and resorted to other more familiar ways of marking halves in a square, as in Fig. 2.

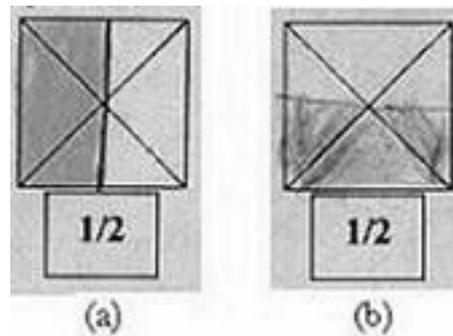


Fig. 2. Students' answers may not take into account all the given elements in a shape.

Based on this type of analysis, performed on the entire data corpus from the problem-solving sessions, we came up with an illustrative list of conceptual difficulties, to be addressed in the design process of a tangible artifact aimed at supporting the teaching and learning of fractions:

- Understanding the notion of “equal parts” and partitioning areas (or even volumes) that require a more accurate spatial reasoning.
- Exploring and recognizing multiple ways of partitioning areas, other than the more common ones.

4.5 Tests with a prototype

We then developed a low-fidelity prototype of a product intended to help students to deal with divisions of geometric figures, specially in relation to the conceptual

difficulties identified above. Our prototype consisted of geometric figures cut in white paper and a set of blue-colored paper sticks. The sticks were supposed to work as dividing lines in figures. The students could therefore manipulate the sticks freely, placing them on top of the figures to try and find the desired divisions. Fig. 3 shows a child's attempts to divide an hexagon in three equal parts.

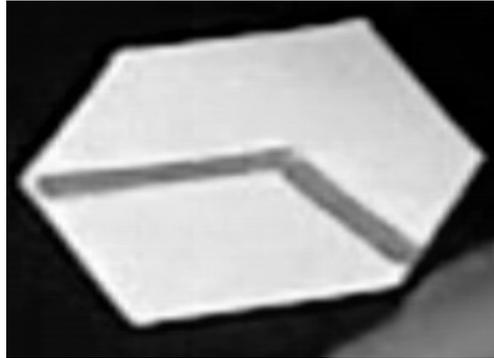


Fig. 3. Prototype made out of paper and paper sticks

We noticed that the students were engaged and interested in the activities with the prototype. They used it easily and were able to explore a greater number of possible ways of partitioning figures than when using paper and pencil only. These materials stimulated collaborative work and interactions, making the students observe each other's actions. The product also made students' actions explicit for us as researchers, helping in the way we delivered hints and explanations. This prototyping phase led us to finalize a product we called *Parts&Bits*, presented in the next section.

5 *Parts&Bits*

Parts&Bits is a tangible interface designed to help the teaching and learning of fractions, with a focus on fractions as part-whole relationships of continuous areas. As we have seen, students are traditionally taught only the most common visual forms of partitioning figures and therefore have difficulties to deal with uncommon forms and divisions. The aim of *Parts&Bits* is to make students explore and visualize different ways of dividing 2D figures and to help them associating parts to fraction names.

Parts&Bits was designed according to the guidelines presented in Sect. 4.1 of this paper, and also in accord with the requirements elicited by our own research (Sect. 4.3). In this section, we describe *Parts&Bits* indicating which of those

requirements (marked heretofore in italics, and the corresponding requirement number in Sect. 4.3 between parenthesis) are satisfied by each of its features.

The interface is then fairly *simple* (#9):

- straight sticks connectable to each other;
- curve sticks also connectable to each other;
- a board of approximately 30cm x 40cm.

The material employed in the construction of *Parts&Bits* must be *resistant* (#10) and heavy enough to *resist to wind* (#14) (like plastic or wood). At the same time it must be *safe* (#12) for children to use - the sticks must have rounded extremities.

The sticks can be used to form 2D geometric figures on top of the board as well as to divide them. The board is such that can detect the configuration of closed figures, consider them as “wholes”, and calculate the fraction corresponding to each sub-area formed inside them. The board would then automatically illuminate areas of same size with the same color, and use different colors for different sizes (#9 - *simple interface*) (see Fig. 4 (a)).

Besides illuminating the areas, the board would emit a sound for each sub-area, (following the same rule stated for the coloration), making it suitable for use by the *visually impaired* (#8). This functionality could be disabled or regulated (#13 - *audio control*).

The board would also exhibit a representation of the fractions corresponding to each area built on its top (see Fig. 4 (b)). For each new division, the board would recalculate the areas.

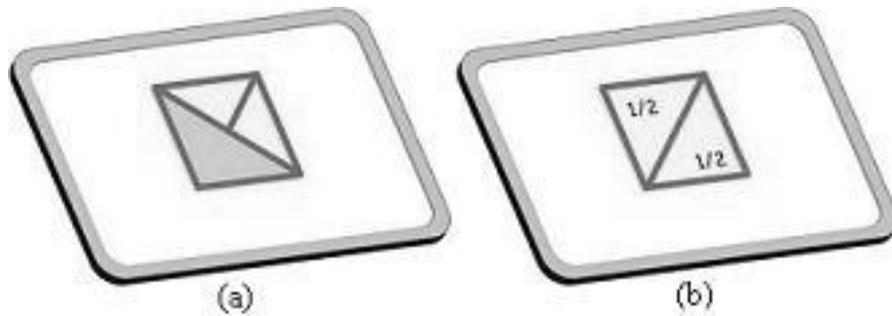


Fig. 4. Tentative illustrations of Parts&Bits

Scaffolding (#1) would therefore be provided by choosing one of these three options: using *Parts&Bits* with no hints, using it along with colors and/or sounds or making it to show the fractions themselves (#2 - *reasoning*), (#5 - *multiple activities*).

Our purpose is that students manipulate the sticks with freedom and flexibility (#2 - *reasoning*), building geometric forms and partitioning them in ways that are conceptually more engaging than using only paper and pencil.

Parts&Bits can be easily *used collaboratively* (#7) by small groups of children, performing *many different activities* (#5), such as:

- free exploration;
- exploring divisions of given geometric figures;
- forming and understanding the concept of “whole” (#4 - *curriculum adequacy*): reasoning about the concept of whole can be made by successively placing sticks as division lines and taking them out, for example;
- equivalence of fractions (#4): by forming subareas of different sizes and making *Parts&Bits* show the corresponding fractions, teachers may use the interface to make students understand notions of equivalence. This process may also help to perform operations of fractions with different denominators, such as $1/2 + 1/4$.

Besides satisfying our requirements, we believe *Parts&Bits* attends to the aspects we stated as fundamental for tangible interfaces in the educational field (Sect. 2). Its format is adequate to classrooms and it is independent of personal computers. The use of diverse media allows access to children with different disabilities. Scaffolding is provided and the interface can be easily used collaboratively. The concepts explored by *Parts&Bits* are part of the curriculum of elementary schools in Brazil and many other countries. Finally, although we cannot guarantee user engagement as no tests were performed with an actual tangible interface, the observations of prototype usage and the report of teachers in the interviews made us believe that students would be very willing to try and learn with *Parts&Bits*.

As for the *low cost requirement* (#11) we depend on studies of implementation, discussed in the next section.

6 Implementation issues

Available low-cost technologies could be used to implement some features of *Parts&Bits*. In what follows we present some aspects of a to be proposal of implementation.

In our proposal, sticks are built on wood or plastic, with magnetic connections at each extremity. An embedded ARM microprocessor is used to calculate and compare areas formed on the board. This microprocessor also displays simple sounds or plays previously recorded sounds.

The feature of detecting the sticks on the board and coloring the areas represent the greatest challenge for implementation of *Parts&Bits*. Coloring and showing numbers on the board could be done through *Light Emitting Diodes* (LEDs), embedded under the board’s surface. LEDs are low-cost, but to produce a good variety of colors we would need RGB LEDs, which are quite expensive.

Using LEDs for coloring means that we would need another technology to detect the sticks. Infrared sensors placed on the sides of the board could not be used, as they wouldn't be able to detect sticks in the interior of figures.

An alternative for detection in this case would be to use a camera and computational vision techniques. However, because of the LEDs placed under the board, the camera would have to be located over the board and this would lead to a fragile product not suitable for prolonged classroom usage. The same kind of problem is faced by MIT's *Sensetable* [20], which tracks objects and displays images through a projector placed over the board.

Also an MIT product, the platform *TViews* [13] provides object tracking and has an embedded graphical display. The tracking is realized through acoustic sensors and propagation in glass. The cost of *TViews* is not high, however the use of glass would go against our safety requirement. Furthermore, the acoustic technology would probably suffer negative influence of noisy classroom environments.

To sum up, we could not comply with all requirements listed for *Parts&Bits* if employing only today's low-cost technologies. On the other hand, some options of moderate cost could be implemented if we relaxed some requirements. Additionally, recent works have explored innovative ways to implement multi-tracking surfaces, and thus showing how minor adaptations could allow full implementation of *Parts&Bits*.

7 Conclusions and Future Work

In this paper, we presented a qualitative piece of research aimed at eliciting guidelines and requirements for the design of educational tangible interfaces. The results supported the proposal of a tangible interface for the learning of fractions, a topic that causes major difficulties for students in elementary math classrooms.

Educational tangible interfaces seek to expand the possibilities offered by traditional manipulatives through technological innovations, enhancing the learning process. To design an useful and viable tangible interface, adequate to Brazilian classrooms and children, we followed a context-based methodology, oriented by Activity Theory and user-centered design guidelines.

Parts&Bits is the tangible interface proposed in our work. Satisfying all requirements and keeping implementation at a low cost have proved to be a challenge. This paper offered a view of such a challenge and, we hope, contributed with paths to its overcoming.

Future research in this area should address issues of innovation, low-cost technologies for implementing *Parts&Bits*, and the consequent development of high-fidelity prototypes for accurate usability tests. Furthermore, the data corpus built during the problem-solving sessions with students may be revisited to generate other proposals of tangible interfaces, exploring yet other approaches to fractions and its learning.

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